# Math 241, Practice Exam 1

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## 1. Exercise

(a) Compute an equation for the unique plane containing the three points

$$P_1 = (2, 3, 4), \quad P_2 = (3, 3, 5), \quad P_3 = (0, 1, -1).$$

(b) Find the symmetric equations for the line perpendicular to the plane

$$3x - 7y + z = 5$$

and passing through the origin.

(c) Find $\cos\theta$, where $\theta$ is the angle between $\mathbf{a} = (\sqrt{2}, -\sqrt{3}, 2)$ and $\mathbf{b} = (\sqrt{2}, \sqrt{3}, 5)$.

(d) Find the distance from the point $P = (1, 1, 1)$ to the plane with equation

$$2x - y + 3z = 1.$$

(e) Write the formula for $(a, b, c) \times (x, y, z)$.

## 2. Exercise

Consider a horizontal lever 10 feet in length attached at one end to an axle. Suppose a force $\mathbf{F}$ of magnitude 20 pounds is applied to the other end of the lever and that $\mathbf{F}$ makes an angle $45^\circ$ with the horizontal.

-Find the magnitude of the moment around the axle.

## 3. Exercise

(a) Compute the integral $\int_1^2 \mathbf{F}(t)\,dt$ where

$$\mathbf{F}(t) = \ln t\mathbf{i} + t^2\mathbf{j} + \cos(\pi t)\mathbf{k}.$$

(b) Find the length of the curve $C$ parametrised by the smooth function

$$\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t^{3/2}\mathbf{k}, \quad 0 \leqslant t \leqslant \frac{20}{3}.$$

## 4. Exercise

Consider a particle with position

$$\mathbf{r}(t) = \frac{t^3}{3}\mathbf{i} + \frac{\sqrt{3}}{\sqrt{2}}t^2\mathbf{j} + 3t\mathbf{k}$$

at time $t$

(a) Compute $(3 + t^2)^2$.

(b) Compute the velocity $\mathbf{v}(t)$.

(c) Compute the acceleration $\mathbf{a}(t)$.

(d) Compute the speed $\|\mathbf{v}(t)\|$.

(e) Find the tangent $\mathbf{T}$.

(f) Find the tangential component $a_T$ of acceleration.

(g) Find the normal component $a_N$ of acceleration.